

## Lesson 3-2: Proving Lines Parallel

### Converses

Last lesson we discovered properties of transversed parallel lines. Given Postulate 3-1 (Corresponding Angles Postulate) we discovered Theorem 3-1 (Alternate Interior Angles Theorem) and Theorem 3-2 (Same-Side Interior Angles Theorem). With these tools, if we knew that transversed lines are parallel, we could draw conclusions about the angles they form.

Today we are going to consider the converse to those tools. In other words, if we know information about the angles, can we conclude the transversed lines are parallel?

### Postulate 3-2 Converse of Corresponding Angles Postulate

If you recall, Postulate 3-1 says that if parallel lines are transversed, then corresponding angles are congruent. Can we form the converse of that? Yes and it is accepted as Postulate 3-2. Here it is; remember you don't want the hypothesis to assume parallel lines...that statement will go in the conclusion:

If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.

### What about the Converse of Theorem 3-1?

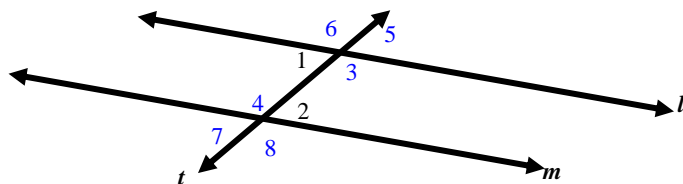
Theorem 3-1 states if two parallel lines are transversed, then alternate interior angles are congruent. The converse would be:

If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

Given that this came from a theorem, we need to prove it.

### Proof

Consider this diagram. It is given that alternate interior angles (let's say  $\angle 1$  and  $\angle 2$ ) are congruent.



We have Postulate 3-2, so if we can prove that corresponding angles are congruent we can conclude the lines are parallel. What is the corresponding angle to  $\angle 2$ ? It is  $\angle 5$ . This works great because  $\angle 1$  &  $\angle 5$  are vertical angles and hence are congruent (Theorem 2-1). We're set.

Given:  $\angle 1 \cong \angle 2$

Prove:  $l \parallel m$

Proof:  $\angle 1 \cong \angle 2$

$\angle 1 \cong \angle 5$

$\angle 2 \cong \angle 5$

$l \parallel m$

Q.E.D.

Given

Vertical  $\angle$ 's are  $\cong$  (Theorem 2-1)

Substitution POC

Corresponding  $\angle$ 's are  $\cong$  (Postulate 3-2)

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### Theorem 3-3 Converse of the Alternate Interior Angles Theorem

If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

### Theorem 3-4 Converse of the Same-Side Interior Angles Theorem

Theorem 3-2 states if two parallel lines are transverses, then same-side interior angles are supplementary. The converse is:

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

This is theorem 3-4. Shall we prove it? Well, we won't here; its proof is one of the problems in the homework assignment. I don't want to ruin that for you! We will just accept it for now.

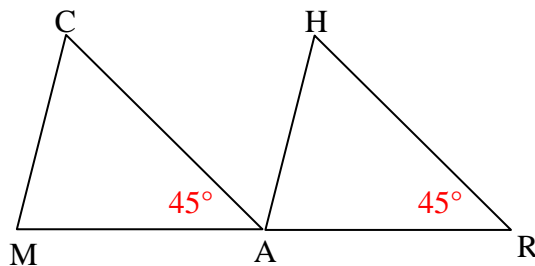
### Putting it all to work

Take a few moments and look through example 2 pg. 125 for a simple way in which we can use this theorem. Once you have it down, consider problem #2 on pg. 125:

Which lines or segments are parallel? Justify your answer with a theorem or postulate.

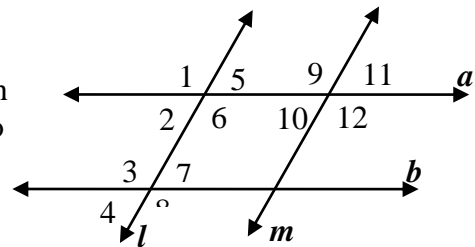
The trick here is to see  $\overline{MR}$  as the transversal. Then since  $\angle CAM$  and  $\angle HRA$  are both  $45^\circ$ , they are congruent. Applying the Converse of Corresponding Angles Postulate, we

have congruent corresponding angles, and thus  $\overline{CA} \parallel \overline{HR}$ .



### ...and again...

Take a look at problems #4-14 even on pg. 125. For each problem determine if the information given allows you to conclude which lines (if any) are parallel. Try to work through them without looking at the answers below.



4.  $\angle 2, \angle 3$  supplementary -  $a \parallel b$ ; converse same-side int angles theorem
6.  $\angle 4, \angle 8$  supplementary - none; adjacent suppl angles not sufficient
8.  $\angle 1 \cong \angle 3$  -  $a \parallel b$ ; converse of corresponding angles post.
10.  $\angle 3 \cong \angle 6$  -  $a \parallel b$ ; converse of alt int angles theorem
12.  $\angle 1 \cong \angle 6$  - none; vertical angles not sufficient
14.  $\angle 11 \cong \angle 7$  - none; would have to assume lines are parallel to make any connections

## Lesson 3-2: Proving Lines Parallel

### Parallel to parallel lines

If you have two parallel lines, and a third line is parallel to one of them, what would you assume? It is likely you said that if it is parallel to one of the parallel lines it is parallel to both. Well, that is the gist of Theorem 3-5.

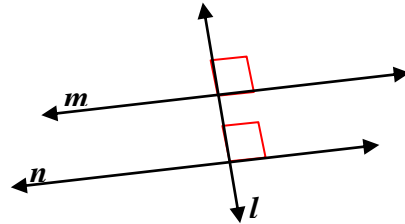
### Theorem 3-5

If two lines are parallel to the same line, then they are parallel to each other.

Again, you may ask about the proof (I do know you love proofs). This is another one we will leave to problem #26. You can't wait, can you?

### Perpendicular lines

Consider the following diagram. If all you were told is that  $m \perp l$  and that  $n \perp l$ , what would you conjecture about lines  $m$  and  $n$ ?



It would certainly be reasonable to assume that  $m \parallel n$ .

Good conjecture; let's prove it. Ha! Thought we'd escape this one didn't you?

### Proof

Well this one is pretty easy. First we can name the top right angle 1 and the bottom 2. Given that both are right angles (definition of perpendicular), they are congruent (Theorem 2-4). Next we consider that line  $l$  transverses lines  $m$  and  $n$ . This makes  $\angle 1$  and  $\angle 2$  corresponding angles and the Converse of the Corresponding Angles Postulate allows us to conclude that  $m \parallel n$ . Q.E.D.

### Theorem 3-6

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

Now, a question: there is a key difference between the wording of Theorem 3-5 and 3-6. Can you spot it? It isn't that one is talking about parallel and the other perpendicular.

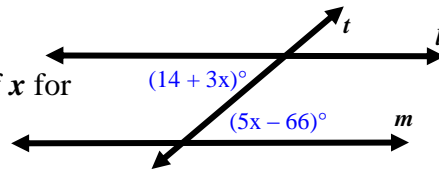
Theorem 3-6 has a very important qualifier: in a plane. Can you think of a counter-example that would prove 3-6 wrong if that qualifier wasn't there?

How about the corner of the room at the ceiling? We have two horizontal lines (the wall/ceiling lines) that are both perpendicular to the same line, the vertical corner (wall/wall line). The problem is the two horizontal lines intersect at the corner. Hence the qualifier "in a plane." This can not happen with coplanar lines. If you doubt me, take a few minutes and try to draw a case with coplanar lines that works.

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### The algebra connection

Consider the following diagram and find the value of  $x$  for which  $l \parallel m$ .



First step is to classify the two angles. They are alternate interior angles. If we assume that  $l \parallel m$ , is there a postulate or theorem we can use to help? Yes there is! By the way, if you don't have a postulate and theorem list right now, shame on you! Theorem 3-1 is the one we want: Alternate Interior Angles Theorem. This means if we have parallel lines and we can find alternate interior angles, we know they are congruent. Thus:

$$\begin{array}{ll} 14 + 3x = 5x - 66 & \text{Alt int } \angle's \cong \\ 80 = 2x & \text{Subtr \& Add POE (+66 both sides, -3x both sides)} \\ 40 = x & \text{Div POE } (\div 2 \text{ both sides)} \end{array}$$

### Assign homework

p. 125 #1-15 odd, 16-23, 26, 27, 29, 33, 35, 47, 51-57